Homework: http://math.mnrt.net/

Name ________________________________ ______ raw ______ scaled ______ percent

Math 11 Trimester 2 Practice Exam 1P

This is the second part of what you worked on weeks ago

- **Partial credit may be given for correct work. Therefore, it is to your advantage to write clear solutions. If I cannot understand a solution within 90 seconds, then it will receive no partial credit.**

- **Answers must be completely simplified. No denominators may include radicals. All fractions reduced. Arithmetic must be completely performed; e.g. write 9 instead of $\sqrt{81}$.**

- **All angles you write for answers must be written with respect to the angle zero and measured in the positive direction (counter clockwise). For example, write $\theta = \frac{3\pi}{2}$ rather than $\theta = -\frac{\pi}{2}$.**

- **No calculators. All answers must be exact.**

D. Proofs

[1] Prove any one of the addition theorems that involves only sines and cosines.

[2] Prove: $\sin 2\alpha = 2\sin \alpha \cos \alpha$

[3] Prove: $\sin \alpha \cos \beta = \frac{1}{2}[\sin (\alpha + \beta) + \sin (\alpha - \beta)]$

[4] Prove: $\cos 2x = 2\cos^2 x - 1$

■ E. Answer the following

[1] Find the maximum and minimum values of $y = \sin\theta + \sqrt{3}\cos\theta$ <u>and</u> state the values of θ, $0 \le \theta < 2\pi$, at which they occur.

[2] Solve for θ, $0 \le \theta \le 2\pi$, if $\mathbf{\sin\theta = 2\ \sin^2\theta}$.

[3] Solve for x, if $\mathbf{5\ \cos 3x - 1 = 4\ \cos 3x + 1}$

[4] Solve for x, if $\mathbf{\cos\ 2x - 1 = 3\ \cos\ 2x - 1}$

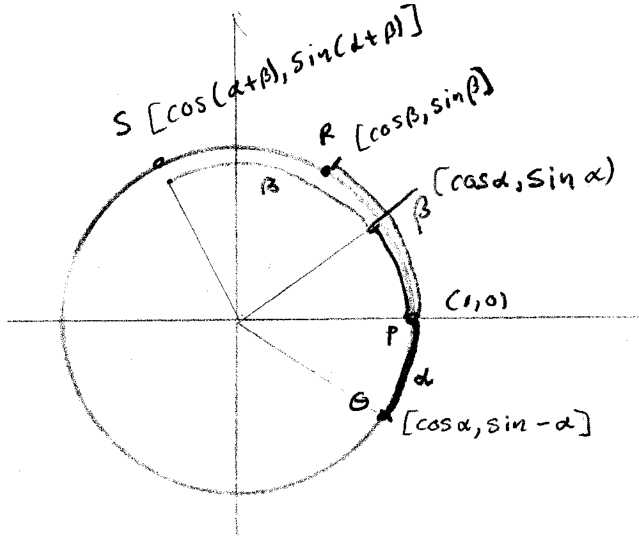
[5] Solve for θ, if $\mathbf{\sin\ 4\theta = -\sin\ 3\theta}$

Answers Math 11 Trimester 2 Practice Exam 1P

Trigonometry

■ D. Proofs

[1] Prove any one of the addition theorems that involves only sines and cosines.



$\overset{\frown}{PS} = \overset{\frown}{QR} \Longrightarrow \overline{PS} = \overline{QR} \Longrightarrow \overline{PS}^2 = \overline{QR}^2$.

$\overline{PS}^2$

$= (\cos(\alpha+\beta) - 1)^2 + (\sin(\alpha+\beta))^2 = \cos^2(\alpha+\beta) - 2\cos(\alpha+\beta) + 1 + \sin^2(\alpha+\beta)$

$= 2 - 2\cos(\alpha+\beta)$

$\overline{QR}^2$

$= (\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin(-\alpha))^2$

$= (\cos\beta - \cos\alpha)^2 + (\sin\beta + \sin\alpha)^2$

$= \cos^2\beta - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\beta + 2\sin\alpha\sin\beta + \sin^2\alpha$

$= 2 - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta$

$\overline{PS}^2 = \overline{QR}^2 \Longrightarrow \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

□

[2] Prove: $\mathbf{\sin 2\alpha = 2\sin\alpha\cos\alpha}$

$\sin(2\alpha) = \sin(\alpha + \alpha) = \sin\alpha\cos\alpha + \sin\alpha\cos\alpha = 2\sin\alpha\cos\alpha$ □

[3] Prove: $\mathbf{\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$ (EQ1)

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$ (EQ2)

$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$ (EQ1) + (EQ2)

$\therefore \sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

□

[4] Prove: $\mathbf{\cos 2x = 2\cos^2 x - 1}$

$\cos 2x = \cos(x+x) = \cos x\cos x - \sin x\sin x = \cos^2 x - \sin^2 x$

$= \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$ □

■ E. Answer the following

[1] Find the maximum and minimum values of $\boldsymbol{y = \sin\theta + \sqrt{3}\cos\theta}$ and state the values of θ, $0 \le \theta < 2\pi$, at which they occur.

Let point P in the coordinate plane have coordinates $P(1, \sqrt{3})$. Then distance origin to P is $\overline{OP} = \sqrt{1^1 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$. If $\overline{OP}$ makes angle α with positive x-axis, then

$\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\alpha = \frac{1}{2}$.

$\sin\theta + \sqrt{3}\cos\theta = 2\left(\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right) = 2(\cos\alpha\sin\theta + \sin\alpha\cos\theta) = 2\sin(\alpha+\theta)$.

Now, $\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} \Longrightarrow \alpha = \tan^1\sqrt{3} \Longrightarrow \alpha = \frac{\pi}{3}$.

Thus, $2\sin(\alpha+\theta) = 2\sin(\theta + \frac{\pi}{3}) = y$.

$\therefore\ y_{\max} = 2$ when $\theta + \frac{\pi}{3} = 1 \Longrightarrow \theta + \frac{\pi}{3} = \frac{\pi}{2} \Longrightarrow \theta = \frac{\pi}{6}$.

$\therefore\ y_{\min} = -2$ when $\theta + \frac{\pi}{3} = -1 \Longrightarrow \theta + \frac{\pi}{3} = \frac{3\pi}{2} \Longrightarrow \theta = \frac{7\pi}{6}$.

[2] Solve for θ, $0 \le \theta \le 2\pi$, if $\boldsymbol{\sin\theta = 2\sin^2\theta}$.

$2\sin^2\theta - \sin\theta = 0 \Longrightarrow \sin\theta\,(2\sin\theta - 1) = 0 \Longrightarrow \theta = \frac{\pi}{2}$ or

$\sin\theta = 0 \Longrightarrow \theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

$2\sin\theta - 1 = 0 \Longrightarrow \sin\theta = \frac{1}{2} \Longrightarrow \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$.

[3] Solve for x, if $\boldsymbol{5\cos 3x - 1 = 4\cos 3x + 1}$

$5\cos 3x - 1 = 4\cos 3x + 1 \Longrightarrow \cos(3x) = 2 \Longrightarrow$ no solution.

[4] Solve for x, if $\boldsymbol{\cos 2x - 1 = 3\cos 2x - 1}$

$2\,\mathrm{Cos}[2x] = 0 \Longrightarrow \cos[2x]\ 0 \Longrightarrow 2x = \frac{\pi}{2}$ or $2x = \frac{3\pi}{2}$

Thus, $S = \left\{x \ni x = \frac{\pi}{2} + 2n\pi,\ n \in \mathbb{Z}\right\} \bigcup \left\{x \ni x = \frac{3\pi}{2} + 2n\pi,\ n \in \mathbb{Z}\right\}$

[5] Solve for θ, if $\boldsymbol{\sin 4\theta = -\sin 3\theta}$

$\sin 4\theta = -\sin 3\theta \Longleftrightarrow \sin 4\theta = \sin -3\theta$

$\Longleftrightarrow 4\theta = -3\theta + 2n\pi$ **or** $4\theta = \pi + 3\theta + 2n\pi$

$\Longleftrightarrow \theta = \frac{-2n\pi}{7}$ **or** $\theta = \pi + 2n\pi = (2n - 1)\pi$, where $n \in \mathbb{Z}$